Technische Universität München **Winter99/2000**
Institut für Informatik
Data Compression
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Problem Set Two
Due: February 22, 2000

Problem 1 [Sayood, number 1, page 353]

A square matrix A has the property that $A^TA = AA^T = I$, where I is the identity matrix. If X_1 and X_2 are two N-dimensional vectors and

$$\Theta_1 = AX_1$$

$$\Theta_2 = AX_2$$

then show that

$$|X_1 - X_2|^2 = |\Theta_1 - \Theta_2|^2.$$

Problem 2 [Sayood, number 2, page 353]

Consider the following sequence of values:

$$\begin{array}{rrrrrrrr} 10 & 11 & 12 & 11 & 12 & 13 & 12 & 11 \\ 10 & -10 & 8 & -7 & 8 & -8 & 7 & -7 \end{array}$$

a) Transform each row separately using an eight-point DCT.

b) Combine all 16 numbers into a single vector and transform it using a 16-point DCT.

c) Compare the results of a) and b). For this particular case would you suggest a block size of 8 or 16 for greater compression. Justify your answer.

Problem 3 [Sayood, number 3, pages 353-4]

Consider the following "image":

$$\begin{array}{cccc} 4 & 3 & 2 & 1 \\ 3 & 2 & 1 & 1 \\ 2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{array}$$

a) Obtain the two dimensional DWHT transform by first taking the one-dimensional transform of the rows, then taking the column-by-column transform of the resulting matrix.

b) Obtain the two-dimensional DWHT transform by first taking the one-dimensional transform of the columns, then taking the row-by-row transform of the resulting matrix.

c) Compare and comment on the results of a) and b).

Problem 4

Recall that with the Singular Value Decomposition (SVD) transform, an $n \times n$ matrix X of pixel values can be compressed into at most n nonzero values, called the singular values of X. More precisely, the compression is based on the fact that X can be decomposed in the following fashion:

$$X = U \Lambda V^T$$

where the columns of U and V are the eigenvectors of XX^T and X^TX respectively, and Λ is an $n \times n$ diagonal matrix containing the singular values of X along the diagonal of Λ.

$$\Lambda = U^T X V \text{ is the forward transform of } X$$

and to decompress X we use:

$$X = U \Lambda V^T \text{ which is the inverse transform.}$$

Use the SVD transform to compress the following pixel image:

$$X = \begin{bmatrix} 0 & 1 & 2 & 1 & 0 \\ 1 & 3 & 4 & 3 & 1 \\ 2 & 4 & 5 & 4 & 2 \\ 1 & 3 & 4 & 3 & 1 \\ 0 & 1 & 2 & 1 & 0 \end{bmatrix}$$

Problem 5

Consider the Adaptive Huffman Coding Algorithm. Assume that the source characters consist of the 26 lowercase letters of the English alphabet.

Show how the tree develops when the sequence of characters in "pfaffenhofen" is transmitted.

Problem 6

Use the Burrows-Wheeler algorithm to compress "pfaffenhofen".

Problem 7 [Salomon, number 1.4, page 14]

Given the following 8×8 bitmap, use RLE to compress it, first row by row, then column by column, then in zigzag order. Note that 0 denotes the white color and 1 the black color.

$$\begin{matrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{matrix}$$

a) Show the encoded stream.

b) Compress the bitmap using the Quadtree compression algorithm.

b) Which compression scheme produces the best result?

Problem 8

Consider the following grayscale "image":

$$\begin{matrix} 20 & 30 & 20 & 30 \\ 50 & 40 & 30 & 20 \\ 80 & 40 & 50 & 60 \\ 30 & 40 & 40 & 10 \end{matrix}$$

a) Compress the image using the original 4×4 BTC algorithm with a two-level quantizer. Show the encoded stream. Recall that the output stream is of the form: $\langle$ mean, standard deviation, ...$\rangle$.

b) Decompress the encoded stream.

c) Compress the image using the modified BTC, where the encoder outputs the result in the following form: $\langle$ hight, low, ...$\rangle$.

d) Decompress the encoded stream obtained in part c).

Problem 9

Use the Dynamic Window-Based RLE with a window range of 4 to encode the following sequence:

72 74 70 68 65 72 70

Problem 10

Consider the following 8 bit/pixel grayscale "image":

16	4	2	1
4	2	8	4
16	4	2	1
4	2	8	4

a) Construct the residual image using the lossless JPEG prediction scheme Î(i,j) = I(i-1,j).

b) Construct the residual image using the lossless JPEG prediction scheme Î(i,j) = I(i-1,j) + 0.5*(I(i,j-1) - I(i-1,j-1)).

c) Let *compression ratio* = *number of of bits per pixel in the original image/number of bits per pixel in the encoded image.*
Which scheme gives better compression results?

Problem 11

The Manhattan metric $d(\vec{x}, \vec{y})$ of two vectors $\vec{x} = (x_1, x_2, \ldots, x_n)$ and $\vec{y} = (y_1, y_2, \ldots, y_n)$ is $d(\vec{x}, \vec{y}) = \sum_{i=1}^{n} |x_i - y_i|$.

We are given the following audio signal:

13, 17, 6, 18, 16, 14, 10, 9, 19, 15, 16, 19

that we would like to compress using vector quantization technique.

a) Use the following codebook and the Manhattan metric to compress the given audio signal. Show the compressed sequence.

Index	Sequence		
0	23	11	16
1	14	18	19
2	10	14	18
3	15	11	7
4	17	23	18
5	12	8	16

b) Same question as part a), except that we now have the following binary tree representation of the codebook, where we assume that left branches are coded with zeros and right branches with ones.