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Department of Computer Science
Chair for Efficient Algorithms
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WS 1999/2000
Problem Set 1
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Efficient Algorithms and Data Structures

Problem 1 (6 credits)

We define the relation $\approx$ on the set of all running-time functions: $f(n) \approx g(n)$ if $f(n)$ is $\Theta(g(n))$.
Show that $\approx$ is an equivalence relation (i.e., a relation that is reflexive, symmetric, and transitive). Note that an equivalence class of $\approx$ consists of all functions with a specified growth rate. In other words, $f(n)$ and $g(n)$ have the same growth rate $\Leftrightarrow f(n) \approx g(n)$.

Problem 2 (6 credits)

Order the growth rates of the following running-time functions, from lowest to highest:

n, 2^n, $n \log_2 n$, $n - n^3 + 7n^5$, $n^2 + \log_2 n$, n^2,
n^3, $\log_2 n$, $\sqrt{n} + \log_2 n$, $(\log_2 n)^2$, $n!$, $\ln n$.

Problem 3 (4 credits)

The scientist Fu Tur is working on a computer that will run 10000 times faster than our present computers. Assuming that Dr. Tur's dream is realized, complete the following table that shows the size of the largest problem instance solvable in one hour for a few growth rates.

Growth Rate	Size of largest problem instance solvable in 1 hour with present computer	Size of largest problem instance solvable in 1 hour with computer 10000 times faster
n	N_1	$10000N_1$
n^2	N_2	
n^5	N_3	
2^n	N_4	
5^n	N_5	

Problem 4 (4 credits)

Show that 2^n is $O(n!)$.

Problem 5 (6 credits)

Determine the growth rate of $\binom{n}{i}$, where i is a fixed positive integer.

Problem 6 (8 credits)

What is the growth rate of $\sum_{i=1}^{n} H_i$, where $H_i = \sum_{j=1}^{i} \frac{1}{j}$ (the i^{th} Harmonic number)?

Hint: One possible solution is to express $\sum_{i=1}^{n} H_i$ in terms of H_n.

Problem 7 (6 credits)

An AVL-tree is defined to be a binary tree that satisfies the property that, for any node X, the depths of the left and right subtrees of X differ by at most one. Let $M(n)$ denote the minimum number of nodes in an AVL-tree of depth n.

a) Show that $M(0) = 1$, $M(1) = 2$, and $M(n) = 1 + M(n-1) + M(n-2)$, for $n \geq 2$.

b) Prove by induction that $M(n) = f_{n+2} - 1$, for all $n \geq 0$, where $f_0 = f_1 = 1$, $f_n = f_{n-1} + f_{n-2}$ are the Fibonacci numbers.