Technische Universität München
Department of Computer Science
Chair for Efficient Algorithms
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WS 1999/2000
Problem Set 10
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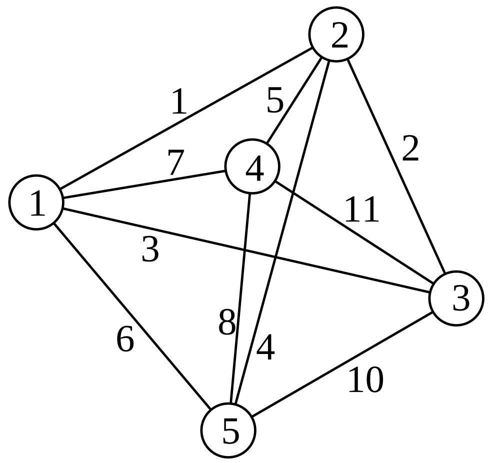
Efficient Algorithms and Data Structures

Problem 1 (12 credits)

Use the dynamic programming algorithm presented in the lecture (it can also be found in D.R. Stinson, *An Introduction to the Design and Analysis of Algorithms*, 1987, pp. 113–117) to solve the following instance of TSP. Recall that the algorithm computes values $P(S,k)$, for $S \subseteq \{2,3,\ldots,n\}$ and $k \in S$, that represent the cost of a shortest path that starts at 1, visits all vertices in $S \setminus \{k\}$, and ends at k. For $|S| > 1$, the value of $P(S,k)$ is calculated as $\min\{P(S \setminus \{k\}, m) + w(m,k) \mid m \in S \setminus \{k\}\}$.

cost matrix: $$\begin{pmatrix} \infty & 1 & 3 & 7 & 6 \\ 1 & \infty & 2 & 5 & 4 \\ 3 & 2 & \infty & 11 & 10 \\ 7 & 5 & 11 & \infty & 8 \\ 6 & 4 & 10 & 8 & \infty \end{pmatrix}$$

graph:



Also, define a $Pred(S,k)$, for each $P(S,k)$, to be the value of "m" which minimizes the expression in calculating $P(S,k)$. Use these values $Pred(S,k)$ to find the optimal Hamiltonian circuit.

Problem 2 (5+5+4 credits)

We describe a dynamic programming algorithm for the minimum tardy task weight problem (cf. Problem 2 of Problem Set 6). Suppose we are given lengths ℓ_i, weights w_i, and deadlines d_i, for $1 \leq i \leq n$. We assume that $d_1 \leq \cdots \leq d_n$, and $\ell_i \leq d_i$, for $1 \leq i \leq n$.

To apply dynamic programming, we need to generalize the problem slightly, by including an overall deadline d, a time by which all tasks must be finished. This has the effect of lowering any deadline $d_i > d$ to d. For $1 \leq j \leq n$, and $0 \leq t \leq d_n$, define $W(j,t)$ to be the minimum tardy task weight obtainable when considering tasks $1,\ldots,j$, with overall deadline t. Given this definition, the solution to the given problem instance is $W(n, d_n)$.

(a) Explain why W satisfies the following recurrence:

$W(j,t) = \min\{W(j-1, d_j - \ell_j), w_j + W(j-1,t)\}$, if $d_j < t$ and $j > 1$

$W(j,t) = \min\{W(j-1, t - \ell_j), w_j + W(j-1,t)\}$, if $l_j \le t \le d_j$ and $j > 1$

$W(j,t) = w_j + W(j-1,t)$, if $t < l_j$ and $j > 1$

The starting conditions are:

$W(1,t) = w_1$, if $\ell_1 > t$

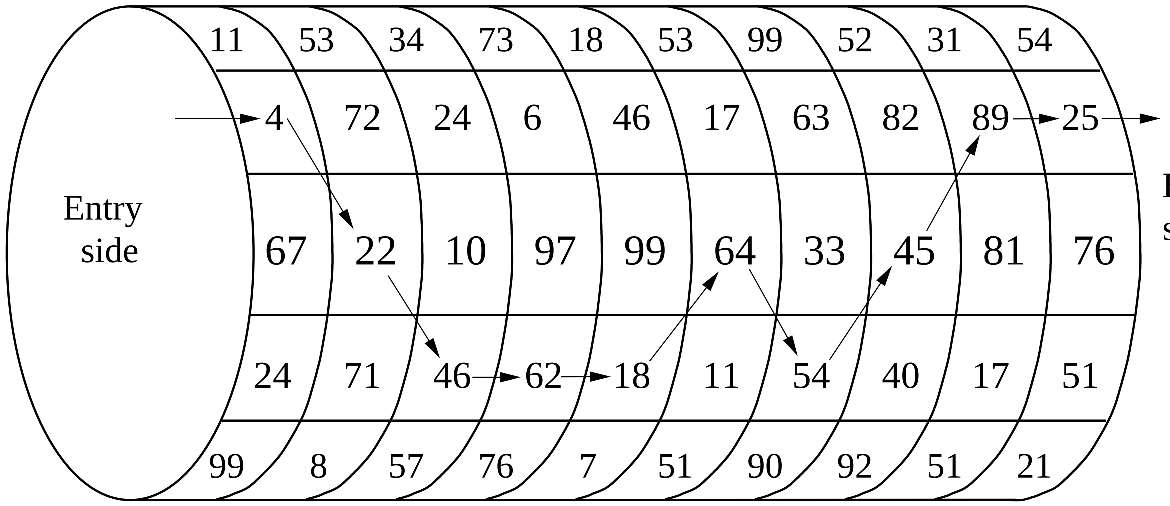
$W(1,t) = 0$, if $\ell_1 \le t$

(b) Using dynamic programming, solve the following instance of this problem:
lengths $1, 5, 3, 2$
deadlines $5, 10, 5, 2$ (resp.)
weights $3, 2, 5, 4$ (resp.)
Note: Calculate the table of $W(j,t)$'s one row at a time.

(c) From the table of $W(j,t)$'s, show how to feasibly schedule the optimal subset.

Problem 3 (4+6+4 credits)

Consider an $n \times n$ array of positive integers (a_{ij}), $0 \le i, j < n$, rolled into a cylinder, as illustrated below.



A path is to be threaded from the entry side of the cylinder to the exit side, subject to the restriction that from a given square it is possible to go only to one of the three positions in the next column adjacent to the current position. The path may begin at any position on the entry side and may end at any position on the exit side. The cost of such a path is the sum of the integers in the squares through which it passes. Thus the cost of the sample path shown above is 429.

(a) How many distinct paths are there?

(b) Show how we can use the dynamic programming technique to find the optimal path (cheapest path).

(c) Find the growth rate of the running time function of your solution to (b).