Technische Universität München
Department of Computer Science
Chair for Efficient Algorithms
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WS 1999/2000
Problem Set 13
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Efficient Algorithms and Data Structures

Problem 1 (4 credits)

Give a simple example of a directed graph with negative-weight edges (but no negative-weight cycles) for which Dijkstra's algorithm produces incorrect answers.

Problem 2 (9 credits)

Consider Dijkstra's algorithm. Suppose we replace the line containing the **while** loop condition:

while $S \neq V$

by:

while $|S| < |V| - 1$

This change causes the **while** loop to execute $|V| - 1$ times instead of $|V|$ times. Is this proposed algorithm correct? Justify your answer.

Problem 3 (9 credits)

We are given a directed graph $G = (V, E)$ on which each edge $(u, v) \in E$ has an associated value $r(u, v)$, which is a real number in the range $0 \leq r(u, v) \leq 1$ that represents the reliability of a communication channel from vertex u to vertex v. We interpret $r(u, v)$ as the probability that the channel from u to v will not fail, and we assume that these probabilities are independent. Give an efficient algorithm to find the most reliable path between two given vertices.

Problem 4 (9 credits)

Modify the Bellman-Ford algorithm so that:

1) it returns TRUE if and only if the graph contains no negative-weight cycles that are reachable from the source s, and returns FALSE otherwise, and
2) it sets $\text{dis}[v] = -\infty$ for all vertices v for which there is a negative-weight cycle on some path from the source s to vertex v.

Problem 5 (9 credits)

Let $G = (V, E)$ be a weighted, directed graph with weight function $w : E \to \mathbb{R}$. Give an $O(|V| \cdot |E|)$-time algorithm to find, for each vertex $v \in V$, the value $\delta^*(v) = \min_{u \in V}\{\delta(u, v)\}$. (Recall that $\delta(u, v)$ is $+\infty$ if there is no path from u to v, is $-\infty$ if there is a path from u to v that contains a negative-weight cycle, and is the weight of a shortest path from u to v otherwise.)