Technische Universität München
Department of Computer Science
Chair for Efficient Algorithms
Prof. Dr. Sami Khuri

WS 1999/2000
Problem Set 14
February 15, 2000

Efficient Algorithms and Data Structures

Problem 1 (10 credits)

Let $G = (V, E)$ be a weighted, directed graph with source vertex s and weight function $d : E \to \mathbb{R}$. Prove that the following claim is true for all vertices $v \in V$: There is a path from s to v if and only if the Bellman-Ford algorithm terminates with $\mathrm{dis}[v] < \infty$ when it is run on G.

Problem 2 (10 credits)

We consider the problem of determining critical paths in PERT chart analysis ("PERT" stands for "program evaluation and review technique"). A PERT chart is a directed, acyclic graph (dag) with weighted vertices. The vertices represent jobs and the edges represent precedence constraints; that is, edge (u, v) indicates that job u must be performed before job v. The weight $w(v)$ of vertex v represents the time that it takes to perform job v. A path through the dag represents a sequence of jobs that must be performed in a particular order. A *critical path* is a longest path through the dag, i.e., a path such that the sum of the weights of the vertices on the path is maximized. Modify the Dag-Shortest-Paths procedure given in the lecture (or in T.H. Cormen, C.E. Leiserson, and R.L. Rivest, *Introduction to algorithms*, p. 536) so that it finds a longest path in a directed, acyclic graph $G = (V, E)$ with weighted vertices in time $O(|V| + |E|)$.

Problem 3 (10 credits)

Give an efficient algorithm to count the total number of paths in a directed acyclic graph. Analyze your algorithm and comment on its practicality.

Problem 4 (10 credits)

Consider Johnson's algorithm for the all-pairs shortest paths problem in sparse graphs (the algorithm was presented in the lecture and can also be found in T.H. Cormen, C.E. Leiserson, and R.L. Rivest, *Introduction to algorithms*, p. 569).

a) What is the purpose of adding the new vertex s to V, yielding V'?

b) Suppose that $w(u, v) \geq 0$ for all edges $(u, v) \in E$. What is the relationship between the weight functions w and $\widehat{w}$?