Technische Universität München
Department of Computer Science
Chair for Efficient Algorithms
Prof. Dr. Sami Khuri

WS 1999/2000
Problem Set 4
November 23, 1999

Efficient Algorithms and Data Structures

Problem 1 (6 credits)

Let A be a sorted array of n distinct integers, some of which may be negative. Design and analyze an efficient algorithm that can find an index k, $1 \leq k \leq n$, such that $A(k) = k$, provided such an index exists. "Analyze" means give the growth rate with justification.

Problem 2 (6 credits)

Rather than splitting $A(1), \ldots, A(n)$ into two subarrays for the purpose of merge-sorting it, let us choose to separate it into three arrays, sort each recursively, and then merge the three sorted arrays. Give a more formal description of this algorithm and analyze its execution time. (You can assume for the sake of simplicity that n is a power of 3.)

Problem 3 (8 credits)

Let $A(1), \ldots, A(n)$ be an array of n integers. An integer is a *majority element* in A if it appears strictly more than $\frac{n}{2}$ times in A. Give an efficient algorithm that can decide whether an array A contains a majority element, and can find it (if A contains one). Analyze your algorithm!

Problem 4 (6 credits)

Solve the following recurrence relations:

a) $T(n) - 7T(n-1) + 10T(n-2) = 3^n, \quad T(0) = 0, \quad T(1) = 1$

b) $T(n) + 6T(n-1) + 9T(n-2) = 3, \quad T(0) = 0, \quad T(1) = 1$

c) $T(n) + T(n-1) + T(n-2) = 0, \quad T(0) = 0, \quad T(1) = 2$

Problem 5 (10 credits)

a) Determine the particular solution of the following recurrence relation

$$T(n) - 2T(n-1) = \rho(n)$$

where $\rho(n) = 7n$.

b) Repeat part (a) for $\rho(n) = 7n^2$.

c) Determine the particular solution of

$$T(n) - T(n-1) = \rho(n)$$

where $\rho(n) = 7n$.

d) Repeat part (c) for $\rho(n) = 7n^2$.

e) Let

$$C_0T(n) + C_1T(n-1) + \cdots + C_kT(n-k) = \rho(n)$$

be a recurrence relation with a characteristic root 1. Let $\rho(n) = n^t$.
What can be said about the general form of the particular solution $T_\rho(n)$?

Problem 6 (4 credits)

Solve the recurrence relation

$$\begin{aligned} &nT(n) + nT(n-1) - T(n-1) = 2^n \\ &T(0) = 273 \end{aligned}$$