Technische Universität München
Department of Computer Science
Chair for Efficient Algorithms
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WS 1999/2000
Problem Set 9
December 21, 1999

Efficient Algorithms and Data Structures

Note: Solutions for this problem set are due on January 18, 2000.

Problem 1 (10 credits)

(Exercise 21.2-2 in [CLR90, p. 431]) An unordered binomial tree is like a binomial tree, but the order of the children of any node can be arbitrary. Unordered binomial trees can also be defined recursively: U_0 consists of a single node, and U_k consists of two unordered binomial trees U_{k-1} for which the root of one is made into *any* child of the root of the other.

Prove that the unordered binomial tree U_k satisfies the following properties:

1. There are 2^k nodes in U_k.
2. The height of U_k is k.
3. There are exactly $\binom{k}{i}$ nodes at depth i for $i = 0, 1, \ldots, k$.
4. The root has degree k, which is greater than that of any other node. The children of the root are roots of subtrees U_0, U_1,…,U_{k-1} in some order.

Problem 2 (10 credits)

(Exercise 21.2-3 in [CLR90, p. 431]) Show that if only mergeable-heap operations (Make-Heap, Insert, Minimum, Extract-Min, Union) are supported, the maximum degree $D(n)$ in an n-node Fibonacci heap is at most $\lfloor \log_2 n \rfloor$.

Problem 3 (10 credits)

(Exercise 21.3-2 in [CLR90, p. 435]) Justify the $O(1)$ amortized time of the operation Fib-Heap-Decrease-Key using the aggregate method, which was introduced in the lecture as one possible technique for amortized analysis.

Problem 4 (10 credits)

(Exercise 21.4-1 in [CLR90, p. 438]) Professor Pinocchio claims that the height of an n-node Fibonacci heap is $O(\log_2 n)$. Show that the professor is mistaken by exhibiting, for any positive integer n, a sequence of Fibonacci-heap operations that creates a Fibonacci heap consisting of just one tree that is a linear chain of n nodes.

References

[CLR90] Thomas H. Cormen, Charles E. Leiserson, and Ronald L. Rivest. *Introduction to Algorithms*. MIT Press, Cambridge, 1990.