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Chair for Efficient Algorithms
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Problem Set 5
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Efficient Algorithms and Data Structures II

Problem 1 (30 credits)

Let $G = (V, E)$ be a flow network woth source s and sink t, and an integer capacity $c(u, v)$ on each edge $(u, v) \in E$. Let $C = \max\{c(u, v) : \ (u, v) \in E\}$.

a. Argue that a minimum cut of G has capacity at most $C|E|$.

b. For a given number K, show that an augmenting path of capacity at least K can be found in $O(|E|)$ time, if such a path exists.

The following modification of Ford-Fulkerson can be used to compute a maximum flow in G.

Max-Flow-By-Scaling(G, s, t)
1 $C \leftarrow \max\{c(u, v) : \ (u, v) \in E\}$
2 initialise flow f to 0
3 $K \leftarrow 2^{\lfloor \log C \rfloor}$
4 **while** $K \geq 1$ **do**
5   **while** there exists an augmenting path p with capacity at least K **do**
6   augment flow f along p
7   **done**
8     $K \leftarrow K/2$
9 **done**
10 **return** f

c. Argue that Max-Flow-By-Scaling returns a maximum flow.

d. Show that the residual capacity of a minimum cut of G is at most $2K|E|$ each time line 4 is executed.

e. Argue that the inner **while** loop of lines 5–7 is executed $O(|E|)$ times for each value of K.

f. Conclude that Max-Flow-By-Scaling can be implemented to run in $O(E^2 \log C)$ time.

Problem 2 (10 credits)

Which of the following statements are true and which are false? Justify your answers by providing either a proof or a counterexample.

a. If f is a maximum flow, then either $f(e) = 0$ or $f(e) = c(e)$ for every edge $e \in E$ of a network $G = (V, E)$ ($c(e)$ denotes the capacity of edge e).

b. For every network there is a maximum flow f, for which either $f(e) = 0$ or $f(e) = c(e)$ for every edge $e \in E$.

c. If the capacities of all edges of a network are pairwise disjoint, then there is a unique minimum cut.

d. If in a network every directed edge is replaced with an undirected one, then the value of the maximum flow remains the same.

e. If you add (multiply) a positive number λ to the capacity of every edge, then the value of a minimum cut remains the same.