Technische Universität München
Department of Computer Science
Chair for Efficient Algorithms
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SS 2000
Problem Set 7
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Efficient Algorithms and Data Structures II

Problem 1 (10 credits)

Five men and five women are attending a dance. Katie will dance only with Garry or Harry, Barbara will dance only with Fred or Ignatz, Carol will dance only with Harry or Jack, Donna will dance only with Fred or Gary, and Emma will dance only with Gary or Ignatz.
Is it possible for all ten people to dance the last dance so that each woman dances with someone she finds acceptable?
[From Graph Theory and its Applications by J. Gross & J. Yellen]

Problem 2 (10 credits)

Suppose that there are four jobs to be performed and five workers who are qualified for every job. The time in hours each worker needs to do each job is given in the table below. Is it possible to assign each job to a different worker so that each worker starts at the same time, and all four jobs are completed within four hours?

	Job1	Job2	Job3	Job4
W1	3	7	5	8
W2	6	3	2	3
W3	3	5	8	6
W4	5	8	6	4
W5	6	5	7	3

[From Graph Theory and its Applications by J. Gross & J. Yellen]

Problem 3 (20 credits)

Suppose that in a given town there are n girls and n boys. Each girl and each boy want to get married, so they rank the members of the opposite sex according to their preference. They submit their preference list to the matchmaker, who then decides who is to marry whom, and finally a town-wide wedding takes place.

A marriage is said to be *unstable* if there is at least one girl and one boy (call them Alice and Bob) such that Alice prefers Bob to her fiance and Bob prefers Alice to his fiancee. In such a case, Alice and Bob will have a tendency to leave their current mates to get engaged to each other. We call the pair Alice-Bob a *dissatisfied pair* under this marriage. A marriage in which there are no dissatisfied pairs is called a *stable marriage*.

In this problem, we describe a greedy algorithm that the matchmaker runs to guarantee that the resulting marriage is stable.

a) Consider the following sketch for the matchmaker algorithm, also called "boy proposes, girl disposes" proposal scheme:
While there are still boys who are not engaged, pick an unattached boy (call him Bob). Bob then proposes to the most desirable girl on his list who has not already rejected him (call this girl Alice). Alice will accept Bob's proposal if she is currently unattached or if her current mate (say Fred) is less desirable to her than Bob. In the latter case, poor Fred is jilted and he reverts back to the unattached state.
This process continues until all the boys get engaged.
Prove that the above algorithm terminates and outputs a marriage.

b) For the algorithm described above, prove that the marriage that results from it is stable.

c) Suppose that the matchmaker now uses the same algorithm but switches around the roles of the boys and girls, so that now it is the girls who propose marriage to the boys.
Show, by giving a counterexample, that this new scheme will not always give the same marriage as the one described in part a.

[From MIT: Goldwasser & Micali]