Dr. Christian Scheideler
Johns Hopkins University

Baltimore, Oct. 19 2004

Theory of Network Communication

Fall 2004

Midterm Exam

Remember:

- Do *not* try to solve everything! There are more problems than you can handle. So stay calm. Start with the problems you like most.
- It is *not* sufficient to cite results in the lecture notes or assignments in order to solve a problem (i.e. statements like "the rest is the same as in the proof of Theorem 3.2" are not allowed, but you are certainly allowed to copy any statements made in that proof into your answers). Every answer to a problem has to be self-contained.

Problem 1: Basic Understanding (5 points)
Please answer these questions in words and/or pictures (*not* formulas; a paragraph for each question is fine).

(a) What does the expansion of a network measure, and why is it good to have a high expansion?

(b) Is the tree a good network for permutation routing? Justify your answer.

(c) Why is it in general not a good idea to use a path system for oblivious routing that has only a single path for every source-destination pair?

(d) What is the difference between hashing and caching?

(e) Name one oblivious and one non-oblivious hashing strategy, and justify your answer.

Problem 2: Connectivity and Diameter (5 points)
For any two binary sequences $x, y \in \{0,1\}^d$, we define the *Hamming distance* $H(x,y)$ of x and y as

$$H(x,y) = \sum_{i=1}^{d} |x_i - y_i|$$

where x_i (resp. y_i) is the ith bit in x (resp. y). For any $d \geq 4$, let $X(d) = (V, E)$ be an undirected graph with node set $V = \{0,1\}^d$ that has an edge between two nodes $x, y \in \{0,1\}^d$ if and only if $H(x,y) = 2$, and let $Y(d) = (V, E')$ be an undirected graph with node set $V = \{0,1\}^d$ that has an edge between two nodes $x, y \in \{0,1\}^d$ if and only if $H(x,y) = 3$.

(a) Show that $X(d)$ is not connected for every $d \geq 4$. (Hint: draw a picture for $d = 4$ to get an intuition.) (2 points)

(b) Show that $Y(d)$ is connected for every $d \geq 4$. (Hint: for some source-destination pairs you may need to flip some bits more than once, but it works.) (2 points)

(c) Compute the diameter of $Y(d)$ (up to an additive constant). (1 point)

Problem 3: Expansion (2 points)
Compute the expansion of a d-dimensional butterfly. It is sufficient here to guess the right set U and compute the value $c(U, \bar{U})/\min\{c(U), c(\bar{U})\}$.

Problem 4: Flow Number (4 points)
Consider the network in Figure 1. Compute the flow number of this network under the assumption that every edge has a capacity of 1. (Hint: use shortest paths. For each node w of distance 2 from a node v, use two fractional paths from v to w: one along the short-cut edge of v, and one along the hexagon. At the end, every edge should have the same load.)

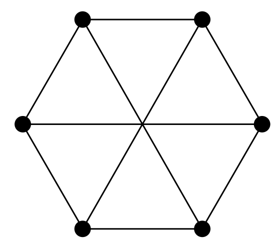

Figure 1: The wheel network.

Problem 5: Universal Stability (4 points)
Our goal will be to show that FTG is universally stable. For this, consider any (w, λ)-bounded adversary with $\lambda = 1 - \epsilon$ where $\epsilon > 0$. Let D be the maximal path length selected by the adversary. We will prove that every packet needs at most

$$\sum_{j=d}^{D-1} \frac{w+1}{\epsilon^{D-j}}$$

time steps to get to a node at which it only has d further edges to traverse. Suppose that this is not true. Let P be the first packet violating this bound. Then it must have an edge e with i remaining edges (including e) to go at which it was delayed at least $(w+1)/\epsilon^{D-(i-1)}$ times (because otherwise the time bound would not be violated). Let e be the first such edge. Then use the approach behind NTO to construct a time sequence I in which there are packets with at least i edges to go that are waiting to cross e. Argue that I must have a size of at least $(w+1)/\epsilon^{D-(i-1)}$ and use a proof similar to the stability proof for LIS to show that this cannot be true.

Good luck!